# Interbank Markets, Foreign Banks, and Monetary Policy Under a Floor System<sup>\*</sup>

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#### Abstract

This paper investigates the role of foreign banks in interbank markets and its implications for U.S. monetary policy under a floor system. We develop a two-country, two-sector banking model to illustrate the behavior of domestic banks, U.S. branches of foreign banks, and other financial institutions in interbank markets. We find that a central bank's balance sheet expansion can increase foreign banks' reserve holdings and their activity in interbank markets, while reducing welfare for both domestic and foreign consumers. In contrast, introducing a reverse repo facility reduces foreign banks' interbank market participation and improves welfare globally.

**Keywords**: Interbank markets; Floor system; Interest on reserves; Foreign banks; Overnight reverse repurchase agreement facility

JEL Codes: E4; E5

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### 1 Introduction

The U.S. federal funds (or fed funds) market serves as an overnight lending platform where depository institutions and government-sponsored enterprises (GSEs) engage in borrowing and lending central bank reserves. Prior to the global financial crisis, this market played a crucial role in redistributing reserve balances throughout the banking system. Institutions that needed funds for clearing, settlement, or to meet reserve requirements would borrow from those holding excess reserves. In this environment, the Federal Reserve intervened in the market for repurchase agreements (the repo market) on a daily basis to maintain its target for the interest rate on fed funds (or the fed funds rate).

In response to the gobal financial crisis, the Federal Reserve launched a large-scale asset purchase program, which necessitated the adoption of a *floor system* for monetary policy operations. Under this system, a sufficiently large quantity of reserves are held in the banking system, reducing the need for institutions to borrow fed funds for regular purposes. The Fed also began setting a nonzero interest on reserve balances (IORB) rate. In principle, arbitrage would make the fed funds rate aligned with the IORB rate, reducing the necessity for frequent Fed interventions in the market.

Considering how monetary policy works in this new environment, the role of foreign banks has become more important than ever before. As depicted in Figure 1, the U.S. branches and agencies of foreign banks significantly increased their reserve holdings compared to domestic banks, resulting in a substantial rise in their share of total reserves from 17.8 percent to 50.2 percent between the first quarter of 2008 and the third quarter of 2014.<sup>1</sup> Moreover, during the same period, the ratio of reserves to total assets for foreign banks surged from 4.1 percent to 56.2 percent, whereas the ratio for domestic banks only rose from 2.7 percent to 11.9 percent. Importantly, foreign banks and GSEs emerged as the primary participants in the fed funds market, as the participation of domestic banks declined.<sup>2</sup> As foreign banks have become increasingly sensitive to changes in monetary policy and rely heavily on wholesale funding from abroad, the Fed's policy interventions can have significant spillover effects on the global financial system, which can also alter the policy effects on the domestic economy.<sup>3</sup>

How do changes in monetary policies—changes in the IORB rate and the size of the Fed's balance sheet—affect other interest rates and welfare in this new environment? Considering the role of foreign banks in allocating resources across countries, what are the implications of U.S. mone-

<sup>&</sup>lt;sup>1</sup>We can observe a similar pattern when the Fed made large-scale asset purchases in response to the COVID-19 pandemic. The share of reserves held by foreign banks initially declined from 34.2 percent in 2019Q4 to 24.2 percent in 2020Q3, and then increased to 42.6 percent by 2023Q1.

<sup>&</sup>lt;sup>2</sup>Afonso, Cisternas, Gowen, Miu, and Younger (2023a) demonstrate that prior to the global financial crisis, the daily trading volume in the fed funds market ranged from \$150 billion to \$175 billion. However, driven by domestic banks, this volume decreased to approximately \$60 billion to \$80 billion in the 2010s. Furthermore, they show that between 65 percent and 95 percent of borrowings have been extended to foreign banks, while more than 90 percent of lendings have been offered by GSEs since 2016.

<sup>&</sup>lt;sup>3</sup>Stern (2022) demonstrates that foreign banks are more responsive to changes in U.S. monetary policy than domestic banks, emphasizing their crucial role in the transmission of policy effects. Relatedly, Kreicher, McCauley, and McGuire (2014) argue that foreign banks' dependence on large volumes of wholesale funding from abroad could make them vulnerable to runs, potentially leading to financial instability.

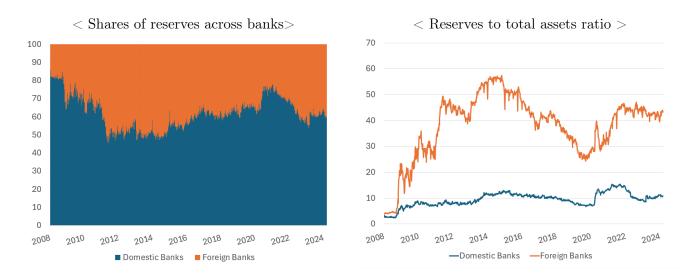


Figure 1: Shares of reserves across banks (left) and reserves to total assets ratio (right)

#### Source: Federal Reserve Board

tary policies for other countries? To answer these questions, we develop a two-country, two-sector general equilibrium banking model where domestic banks, foreign banks, and non-bank financial intermediaries can interact in interbank markets. In this model, banks provide traditional banking services to domestic consumers, while non-bank financial intermediaries offer financial services to both domestic and foreign consumers.

To account for the characteristics observed in the fed funds market, we introduce institutional differences among financial intermediaries into our model. First of all, a legal restriction permits only banks (depository institutions) to earn interest on reserve balances. Despite this restriction, arbitrage would still work in closing any gap between the IORB rate and other overnight rates. In theory, if the IORB rate were to exceed other overnight rates, banks would borrow funds to earn an essentially risk-free interest margin, thereby eliminating the gap. However, this was not the case as illustrated by Figure 2. Since 2008, the IORB rate has consistently exceeded the fed funds rate for the most part.

The lack of sufficient arbitrage arises in part from costs, "balance sheet costs", that banks incur when expanding their balance sheets. These costs include increased deposit insurance premiums and binding capital requirements such as leverage ratios (Bech and Klee, 2011; Afonso, Entz, and LeSueur, 2013; Ennis, 2018; Williamson, 2019). Moreover, there is heterogeneity in balance sheet costs across banks. For example, many foreign banks operating in the United States are not insured by the Federal Deposit Insurance Corporation (FDIC), and therefore, their insurance premiums do not increase with the size of their balance sheets. Also, variations in leverage ratio calculations across jurisdictions make it less costly for foreign banks to expand their balance sheets.<sup>4</sup> Smaller burden on balance sheet costs enables foreign banks to exploit arbitrage opportunities in the fed

<sup>&</sup>lt;sup>4</sup>Specifically, U.S. banks must adhere to leverage ratios based on daily or weekly averages, whereas some foreign banks are only required to meet leverage ratios at the end of each reporting period.

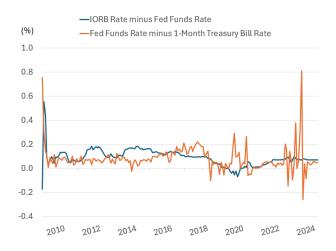


Figure 2: Interest rate spread among short-term interest rates

#### Source: Federal Reserve Economic Data

funds market by increasing their reserve holdings (Kreicher, McCauley, and McGuire, 2014; Afonso, Entz, and LeSueur, 2013; Ennis and Wolman, 2015; Banegas and Tase, 2022).

In the model constructed here, both banks and non-bank financial intermediaries are subject to a limited commitment problem: they can default on their liabilities. This implies that their assets must serve as collateral to prevent potential defaults, imposing balance sheet constraints in the form of collateral constraints. Further, domestic banks in the *home* country, representing the United States, must also meet capital requirements, resulting in tighter balance sheet constraints compared to other financial intermediaries—foreign banks and non-bank financial intermediaries. In equilibrium, a shortage of collateral available to financial intermediaries leads to binding balance sheet constraints for domestic banks and collateral constraints for foreign banks and non-bank financial intermediaries.

Our model demonstrates that under a floor system, balance sheet costs incurred by banks generate an interest rate spread between reserves and interbank lending. When government bonds are sufficiently scarce in the financial system, the collateral constraints of financial intermediaries become binding, resulting in a liquidity premium on collateralizable assets such as government bonds and reserves. A higher liquidity premium implies higher asset prices and lower nominal and real interest rates.<sup>5</sup> In this scenario, non-bank financial intermediaries seek alternative interest-bearing assets to better serve their depositors. If the interbank lending rate falls below the IORB rate, foreign banks are incentivized to engage in arbitrage, while domestic banks remain inactive due to their high balance sheet costs. This leads to an active interbank market where funds flow from non-bank financial intermediaries to foreign banks.

<sup>&</sup>lt;sup>5</sup>The way the model generates an interest rate spread is consistent with Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020), who find that a decrease in the stock of collateral lowered the "repo" rate below the interest rate on reserves. Specifically, they show that purchasing 1% of a bond outstanding implemented by the European Central Bank led to a fall in its repo rate by 0.78 basis points.

Our model also shows that the interbank lending rate can exceed the interest rate on government bonds, consistent with the data depicted in Figure 2. When government bonds become extremely scarce in the financial system, they are exclusively used as liquidity-servicing assets rather than collateral. In this case, reserves cannot serve as substitutes for government bonds due to their limited availability outside the banking system. This scenario arises when the central bank significantly expands its balance sheet, as this expansion involves the exchange of reserves for government bonds. A further increase in the central bank balance sheet increases the liquidity premium on government bonds while reducing the liquidity premium on collateralizable assets. As a result, the interest rate on government bonds falls below the interbank lending rate.

Having identified the conditions that generate active interbank transactions between foreign banks and non-bank financial intermediaries, we can use the model to analyze the effects of monetary policy. Lowering the IORB rate causes a substitution of currency for reserves. That is, the quantity of currency increases in real terms, while the quantity of bank deposits decreases. A lower interest rate on reserves also decreases the redemption value of collateral, thereby tightening collateral constraints. This causes a contraction in foreign bank and non-bank financial sectors, leading to a fall in welfare for foreign consumers.

The effects of a central bank's balance sheet expansion depend on the size of the balance sheet. When the balance sheet is not very large, its expansion can decrease global welfare. The central bank's exchange of reserves for government bonds initially tightens the collateral constraints of nonbank financial intermediaries that are unable to hold reserves. The resulting increase in the liquidity premium on collateralizable assets eventually tightens the balance sheet constraints faced by banks, leading to contractions across all financial sectors and a decline in welfare for both domestic and foreign consumers. Conversely, when the central bank's balance sheet is very large, its expansion can have redistributive effects within the financial system. Given that the market for liquidity-servicing government bonds is effectively segmented from those for collateral, this intervention expands the domestic and foreign bank sectors by increasing the stock of collateral, while causing a contraction in the non-bank financial sector by reducing transactions involving government bonds.

Interestingly, even when a central bank's balance sheet expansion increases the interest rate spread between the interbank lending rate and the IORB rate, foreign consumers can experience a decrease in welfare. While foreign banks earn higher arbitrage profits, which serve to enhance foreign consumers' welfare, this benefit is outweighed by the decrease in welfare due to a tightened collateral constraint. Consequently, foreign consumers' welfare decreases in equilibrium.

We also use this model to study the implications of introducing an overnight reverse repurchase agreement (ON RRP) facility. This facility aims to give the Fed better control over the fed funds rate by allowing non-bank financial intermediaries to hold interest-bearing central bank liabilities. Since foreign banks depend on interbank borrowing to extract arbitrage profits, the ON RRP facility is expected to have a more significant impact on them. We find that, although foreign banks' arbitrage profits decrease, the Fed's ON RRP intervention, which replaces reserves with ON RRPs, helps mitigate balance sheet constraints across the global financial system. This allows financial intermediaries, including banks, to operate more efficiently despite a reduced quantity of reserves. As a result, the policy improves welfare for both domestic and foreign consumers.

Literature Review This paper is related to the empirical literature examining the implications of heterogeneity in bank asset portfolios for monetary policy. Kreicher, McCauley, and McGuire (2014) and Ennis and Wolman (2015) study the effects of the widening of the FDIC assessment base in 2011 on bank asset portfolios. They find that since 2011, foreign banks have played a crucial role in absorbing the increase in reserves in the banking system. Banegas and Tase (2022) show that the revision in the FDIC rule resulted in a 4.5 percentage point increase in the reserves-to-assets ratio of foreign banks relative to domestic banks. Additionally, they find that foreign banks have actively engaged in window dressing by temporarily reducing their reserve holdings, compared to domestic banks, by 18.3 percentage points on reporting days. These empirical papers suggest that the increase in reserves held by foreign banks stemmed from their lower balance sheet costs relative to domestic banks.

This paper contributes to the literature exploring the implications of interbank markets for monetary policy in a closed economy. Papers in this literature include Poole (1968), Ennis and Keister (2008), Afonso and Lagos (2015), Armenter and Lester (2017), Afonso, Armenter, and Lester (2019), Williamson (2019, 2023), Bianchi and Bigio (2022), Afonso, Giannone, La Spada, and Williams (2023b), and Afonso, La Spada, Mertens, and Williams (2023c). Unlike previous studies, our paper specifically focuses on the role of foreign banks in interbank markets and its domestic and global implications for monetary policy.

A related study is that of Stern (2022), who extends Bianchi and Bigio's framework to incorporate foreign banks with lower balance sheet costs. Stern (2022) finds that foreign banks are more responsive to changes in the interest rate on reserves, emphasizing the importance of studying the role of foreign banks in the transmission of monetary policy. With a different focus, our tractable two-country, two-sector banking model allows us to analyze the effects of a central bank balance sheet expansion and its global consequences.

Kim (2023) also develops a two-country model to study how the effects of a central bank's balance sheet expansion can spill over to the global economy. Given that short-term assets are more pledgeable as collateral than long-term assets, Kim (2023) shows that a central bank's swaps of short-term government liabilities for long-term ones can increase the effective stock of collateral in the financial system, ultimately improving global welfare. In the current paper, we focus on replicating the behavior of foreign banks in the financial system by abstracting the different maturities of government liabilities. We find that, mainly due to the legal restrictions on reserve holders, a central bank's swaps of reserves for government bonds can tighten the balance sheet constraints of financial intermediaries and decrease global welfare.

### 2 The Model

There are two countries, *Home* and *Foreign*, representing the United States and the rest of the world, respectively. Each country has three types of domestic agents: *buyers, sellers*, and *banks*. There are also global financial institutions that provide financial services in both countries. For convenience, we call them *mutual funds*. While banks represent typical depository institutions in each country, mutual funds represent all non-depository financial institutions. For each type, there exists a continuum of agents with unit mass. Parameters and variables associated with the Home country are denoted with a subscript h and/or without an asterisk, while their Foreign counterparts are denoted with a subscript f and/or an asterisk. When describing the model environment, we will focus on the perspective of the Home country while keeping in mind that there is a Foreign counterpart.

The model builds on a Rocheteau and Wright (2005) framework, with multiple financial sectors similar to those introduced in Williamson (2019, 2023). Each period is divided into two sequential subperiods—the centralized market (CM) followed by the decentralized market (DM). The CM provides an international platform where all agents from the two countries can interact. Debts acquired in the previous period are settled at the beginning of the CM. Then, Home and Foreign agents produce goods, trade goods for assets, and consume goods. Buyers in each country write contracts with banks and mutual funds at this stage. After the market is closed, buyers can make contact with their banks and mutual funds.

In each country, the government issues three types of assets: currency, reserves, and government bonds. Each country's central bank issues perfectly divisible and portable currency as well as reserves—banks' account balances with the central bank. Currency and reserves issued by the Home country's central bank will be referred to as *Home currency* and *Home reserves*, respectively, and their Foreign counterparts will be referred to as *Foreign currency* and *Foreign reserves*. Home (Foreign) currency sells at price  $\phi$  ( $\phi^*$ ) in the CM. Because agents frictionlessly trade goods and assets in the CM, in equilibrium the nominal exchange rate  $\xi$  (the price of Foreign currency in units of Home currency) satisfies the law of one price, i.e.,  $\phi^* = \xi \phi$ . Each unit of Home (Foreign) reserves sells at price  $z_m$  ( $z_m^*$ ) in the CM and is a claim to one unit of reserves in the following CM. Each country's fiscal authority issues one-period nominal government bonds denominated in the domestic currency. A Home currency-denominated bond (*Home bond*, hereafter) sells at price  $z_b$  in the CM and is a claim to one unit of the Home currency in the following CM. Similarly, a Foreign currency-denominated bond (*Foreign bond*, hereafter) sells at price  $z_b^*$  in the CM and pays off one unit of the Foreign currency in the following CM.<sup>6</sup>

In the CM, buyers can produce goods but cannot consume, whereas sellers can consume goods but cannot produce. Specifically, each buyer incurs disutility H when producing H units of CMgoods, whereas each seller receives utility X from consuming X units of CM goods. Banks and

<sup>&</sup>lt;sup>6</sup>We could also introduce private assets in this model, but this would not change the main results of the paper. Our focus is on which financial intermediaries are authorized to hold a certain type of government liabilities and how these government liabilities are used in financial markets.

mutual funds are active only in the CM, and they can produce and consume goods. Their perperiod preferences are given by X - H, where X and H denote the units of CM goods consumed and produced, respectively.

In contrast to the CM, the DM is a domestic platform where each buyer is randomly matched with a seller within the country, and the buyer makes a take-it-or-leave-it offer to the seller. In the DM, sellers can produce goods but cannot consume, while buyers can consume goods but cannot produce. Each buyer receives utility u(x) from consuming x units of DM goods, whereas each seller incurs disutility h from producing h units of DM goods. The utility function  $u(\cdot)$  is strictly increasing, strictly concave, and twice continuously differentiable with properties  $u'(0) = \infty$ ,  $u'(\infty) = 0$ , and  $-\frac{xu''(x)}{u'(x)} < 1$ . Denote the first-best quantity by  $\hat{x} \in (0, \infty)$  that solves  $u'(\hat{x}) = 1$ .

There is no record keeping device (no memory), and agents cannot be forced to work (limited commitment). This implies that unsecured credit arrangements are infeasible in this economy, and that some assets must be exchanged on the spot or posted as collateral for transactions in the DM. The range of acceptable means of payments in DM transactions varies across sellers. Depending on the seller's ability to evaluate assets, there are essentially four types of sellers in each country. Type c sellers can recognize only paper currency issued by the domestic government, so they accept only domestic currency. Type b sellers can accept those issued by the Foreign country. Finally, type d sellers can evaluate all types of assets such as currencies, government bonds, reserves, and financial intermediary liabilities backed by these assets.

Buyers are divided into two groups: some buyers are bank depositors and the other buyers are mutual fund depositors. Specifically, a fraction  $\alpha$  of buyers in the Home country (*Home buyers*, hereafter) are bank depositors while a fraction  $1 - \alpha$  are mutual fund depositors. Similarly, a fraction  $\alpha^*$  of buyers in the Foreign country (*Foreign buyers*, hereafter) are bank depositors while a fraction  $1 - \alpha^*$  are mutual fund depositors. In the DM, each bank depositor meets a type c seller with probability  $\rho$ , in which case they must carry domestic currency to make a payment. With probability  $1 - \rho$ , the bank depositor meets a type d seller who accepts any types of assets as a means of payment. Mutual fund depositors are matched with a different set of sellers. Specifically, each mutual fund depositor meets a type b seller with probability  $\pi\gamma$ , a type b seller with probability  $\pi(1-\gamma)$ , and a type d seller with probability  $1-\pi$ . Transactions made with type b and type  $b^*$  sellers capture wholesale payments that involve using government bonds, rather than currencies, reserves. or claims on financial intermediaries (banks and mutual funds). When buyers write contracts with financial intermediaries in the CM, they do not know exactly what type of seller they will meet in the following DM. Buyers learn the seller's type after the market shuts down, but they can contact their financial intermediary. Events occuring in the Home country after the CM closes are illustrated in Figure 3.

There are two key assumptions that reflect our observations on financial regulations. First, only a certain set of financial intermediaries are permitted to hold reserve balances with the central bank and earn interest. In the United States, only depository institutions (including the U.S. branches

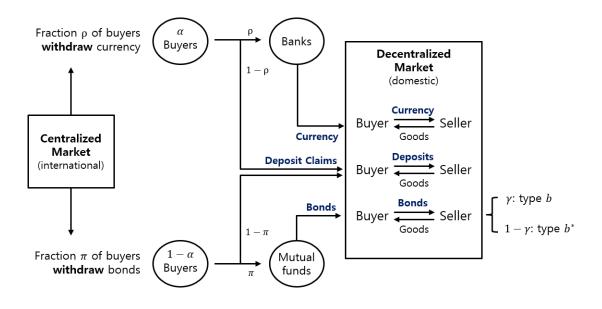


Figure 3: Home country: events after the centralized market closes

of foreign banks) can earn interest on reserve balances with the Federal Reserve.<sup>7</sup> In the model, banks can hold reserve accounts with central banks regardless of their nationality, while mutual funds cannot.<sup>8</sup> Second, some banking sectors are more heavily regulated than others. For instance, traditional banks must comply with tighter financial regulations than other financial institutions. Also, due to the differences in leverage ratio calculations across jurisdictions, U.S. banks have been facing tighter regulations than the U.S. branches of foreign banks. In the model, these observations are represented by a higher balance sheet cost of the banks in the Home country (representing U.S. banks) than those in the Foreign country and mutual funds. Finally, other things equal, assets with higher volatility in their market value typically receive larger haircuts when used as collateral. This feature is reflected in the model that foreign currency-denominated assets.

### 2.1 Fiscal authority and central bank

Confine attention to stationary equilibria where all real variables are constant across periods, and the gross rates of inflation are  $\mu$  and  $\mu^*$  in the Home and Foreign countries, respectively. That is,  $\phi_{t+1}/\phi_t = 1/\mu$  and  $\phi_{t+1}^*/\phi_t^* = 1/\mu^*$  for all t. In each country, the consolidated government

<sup>&</sup>lt;sup>7</sup>This does not necessarily mean that all depository institutions can directly hold reserve accounts with the Fed. According to Ennis and Wolman (2015), there were approximately 7,000 banks operating in the United States in the second quarter of 2011, and around 2,200 banks held reserve accounts with the Fed. Among those, 219 banks were categorized as the U.S. branches and agencies of foreign banks. Approximately 4,800 banks had a small amount of assets (around 5 percent of total assets held by banks), did not directly hold reserve accounts with the Fed, but held their reserves with corresponding banks to earned interest indirectly.

<sup>&</sup>lt;sup>8</sup>There may be institutional differences across countries. For example, not all foreign banks can hold reserve accounts with the Bank of Canada. However, prohibiting U.S banks from holding reserves at the central bank in the Foreign country would not change the main results of the paper.

starts issuing its liabilities with no public debt outstanding at the beginning of period 0. In period 0, the fiscal authority issues government bonds denominated in the domestic currency and makes lump-sum transfers to domestic buyers in equal amounts. The central bank issues its liabilities—the domestic currency and reserves—through open market purchases of government bonds and transfers any profits to the domestic fiscal authority. Consolidated government budget constraints for the two countries in t = 0 are given by:

$$\bar{c} + z_m \bar{m} + z_b b - z_b^* \chi^* = \tau_0,$$
  
$$\bar{c}^* + z_m^* \bar{m}^* + z_b^* \bar{b}^* - z_b \chi = \tau_0^*,$$

where  $\bar{c}$ ,  $\bar{m}$ , and  $\bar{b}$  denote the quantities of Home currency, Home reserves, and Home bonds, in real terms, outstanding at the end of period 0 (and in every following period), and  $\bar{c}^*$ ,  $\bar{m}^*$ , and  $\bar{b}^*$  denote their Foreign counterparts. Also, in real terms,  $\chi^*$  units of Foreign bonds are held by the Home central bank, while  $\chi$  units of Home bonds are held by the Foreign central bank. Finally,  $\tau_0$  and  $\tau_0^*$ denote the real quantities of lump-sum transfers made by the Home and Foreign fiscal authorities, respectively, for t = 0. Consolidated government budget constraints in t = 1, 2, ... are given by:

$$\bar{c} + z_m \bar{m} + z_b \bar{b} - z_b^* \chi^* = \frac{\bar{c} + \bar{m} + b}{\mu} - \frac{\chi^*}{\mu^*} + \tau,$$
$$\bar{c}^* + z_m^* \bar{m}^* + z_b^* \bar{b}^* - z_b \chi = \frac{\bar{c}^* + \bar{m}^* + \bar{b}^*}{\mu^*} - \frac{\chi}{\mu} + \tau^*$$

where  $\tau$  and  $\tau^*$  denote the real quantities of lump-sum transfers (or lump-sum taxes if negative) to each of Home and Foreign buyers for t = 1, 2, ... In the above equations, the left-hand side represents the total value of consolidated government debts issued in each period, net of the value of the assets purchased. The right-hand side represents the total value of debt payments, the payoff from the assets, and the transfers to buyers.

As in Williamson (2019) and Kim (2023), we will assume that the Home and Foreign fiscal authorities comply with the following fiscal rules, respectively,

$$v = \bar{c} + z_m \bar{m} + z_b \bar{b} - z_b^* \chi^*, \tag{1}$$

$$v^* = \bar{c}^* + z_m^* \bar{m}^* + z_b^* \bar{b}^* - z_b \chi.$$
<sup>(2)</sup>

where v and  $v^*$  are exogenously given. That is, each fiscal authority adjusts the quantities of lumpsum transfers for t = 0, 1, 2, ... so as to target the real value of consolidated government liabilities vand  $v^*$ . The balance sheets of the two central banks given fiscal policies v and  $v^*$  are displayed in Table 1.

In practice, there are broadly two different monetary systems adopted by central banks: a *corridor system* and a *floor system*. Under a corridor system, the central bank chooses a short-term interest rate (typically an overnight interest rate) as a policy target and constantly intervenes in the market to achieve its target level. The central bank sets the interest rate on reserves, which

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Assets		Liabilities		Asset	s	Liabilities			
Home bonds	$v - z_b \overline{b}$	Home currency	$\bar{c}$	Foreign bonds	$v^* - z_b^* \overline{b}^*$	Foreign currency	$\bar{c}^*$		
Foreign bonds	$z_b^*\chi^*$	Home reserves	$z_m \bar{m}$	Home bonds	$z_b \chi$	Foreign reserves	$z_m^* \bar{m}^*$		

Table 1: Balance sheets of central banks

serves as the lower bound, and the interest rate on central bank lendings, which serves as the upper bound, to build a corridor for the policy rate. Differently, under a floor system, there is a large amount of reserves issued by the central bank, and the interest rate on reserves plays a more active role in determining the short-term interest rate, rather than being its lower bound.

Our goal here is to replicate the landscape of the U.S. overnight lending market since late 2008. That is, domestic banks are inactive while the U.S. branches of foreign banks are actively participating in arbitrage through borrowing funds at a lower rate, depositing the funds in their reserve accounts, and earning interest at a higher rate. For our purposes, we will assume that the Home central bank operates monetary policy under a floor system, and that the Foreign central bank operates under a corridor system. Specifically, the Home central bank sets the price of Home reserves  $z_m$  (or equivalently, the nominal interest rate on Home reserves  $R_m = \frac{1}{z_m} - 1$ ) and the size of its balance sheet, denoted by  $\omega = \bar{c} + z_m \bar{m}$ . The Foreign central bank determines the price of Foreign bonds  $z_b^*$  and sets the price of Foreign reserves to be higher than this, so that no bank will choose to hold reserves in equilibrium. For simplicity, assume that central banks do not acquire foreign currency-denominated bonds, i.e.,  $\chi^* = \chi = 0$ .

# 3 Characterization of Equilibrium

#### 3.1 Banks and mutual funds

Buyers write deposit contracts with financial intermediaries in the CM before learning the type of means of payment acceptable (or equivalently, the type of seller) in the following DM. In equilibrium, banks in the Home country (*Home banks*, hereafter), banks in the Foreign country (*Foreign banks*), and mutual funds offer deposit contracts to bank depositors in the Home country, bank depositors in the Foreign country, and mutual fund depositors in both countries, respectively. Deposit contracts give each depositor an option either to withdraw a means of payment after learning the type of the seller or trade a claim on deposits, as in Williamson (2019) and Kim (2023).

There are important institutional differences among these financial intermediaries. Home banks and Foreign banks are permitted to hold reserves with the Home central bank, while mutual funds are not. Also, the way the central bank operates varies across countries: the Home central bank operates under a floor system while the Foreign central bank adopts a corridor system. Due to these differences, there arises an interbank market in the Home country where banks and mutual funds can borrow and lend. Suppose that each interbank loan sells at price  $z_e$  in the CM and is a promise to pay one unit of the Home currency in the following CM. Consider a Home bank that issues deposit contracts to bank depositors in the Home country (*Home bank depositors*, hereafter). Since there is a probability that Home bank depositors meet a type c seller who accepts only domestic currency, in equilibrium Home banks will offer a contract that permits a withdrawal of Home currency after the CM closes. If a depositor meets a type d seller who accepts all types of assets, they will not withdraw currency but use their claims on deposits to make payments. Let  $(k_h, c_h, d_h)$  denote the terms of deposit contracts offered by the Home bank, where  $k_h$  is the quantity of goods deposited by each depositor in the CM,  $c_h$  is the quantity of Home currency the depositor can withdraw after the CM closes, and  $d_h$  is the quantity of claims to goods in the following CM for those not withdrawing currency. Also, let  $(m_h, b_h, b_h^*, e_h)$  denote the asset portfolio acquired by the Home bank, where  $m_h$  is the quantity of Home reserves,  $b_h$  is the quantity of Foreign bonds, and  $e_h$  is the quantity of interbank borrowing.<sup>9</sup> All quantities are denoted in terms of current CM goods, except for  $d_h$ , which is denoted in terms of following CM goods.

Then, the Home bank's problem in equilibrium can be expressed as:

$$\max_{k_h, c_h, d_h, m_h, b_h, b_h^*, e_h} \left[ -k_h + \rho u \left( \frac{\beta c_h}{\mu} \right) + (1 - \rho) u \left( \beta d_h \right) \right]$$
(3)

subject to

$$k_h + z_e e_h - \rho c_h - z_m m_h - z_b b_h - z_b^* b_h^* - \beta (1 - \rho) d_h + \frac{\beta (m_h + b_h - e_h)}{\mu} + \frac{\beta b_h^*}{\mu^*} \ge 0,$$
(4)

$$-(1-\rho)d_h - \frac{e_h(1-I_h)}{\mu} - \frac{\delta\rho c_h}{\mu} + \frac{(1-\delta)(m_h + b_h - e_h I_h)}{\mu} + \frac{(1-\theta)(1-\delta)b_h^*}{\mu^*} \ge 0, \quad (5)$$

$$k_h, c_h, d_h, m_h, b_h, b_h^* \ge 0.$$
 (6)

As expressed in the objective function (3), the Home bank must choose a contract  $(k_h, c_h, d_h)$  that maximizes the representative Home bank depositor's expected utility. In each DM meeting, the buyer makes a take-it-or-leave-it offer to the seller. This implies that, in a meeting with a type c seller, the Home bank depositor trades  $c_h$  units of Home currency for  $\frac{\beta c_h}{\mu}$  units of DM goods, whereas in a meeting with a type d seller, they exchange  $d_h$  units of claims to deposits for  $\beta d_h$  units of goods.

Inequality (4) shows that the discounted net payoff for the Home bank must be nonnegative. Specifically, in the CM, the bank receives  $k_h$  units of goods from the depositors and acquires an asset portfolio including Home currency  $\rho c_h$ , Home reserves  $m_h$ , Home bonds  $b_h$ , Foreign bonds  $b_h^*$ , and interbank borrowing  $e_h$ . If the bank borrows on the interbank market, then  $e_h > 0$  and  $I_h = 0$ , whereas if the bank lends on the interbank market, then  $e_h < 0$  and  $I_h = 1$ . After the CM closes, a fraction  $\rho$  of the depositors learn they will meet a type c seller in the DM, so each of them withdraws  $c_h$  units of Home currency. The remaining fraction  $1 - \rho$  of the depositors do not

<sup>&</sup>lt;sup>9</sup>The Home bank always weakly prefers interest-yielding Foreign bonds to zero-interest Foreign currency. Therefore, the Home bank does not hold Foreign currency in equilibrium.

withdraw currency, and in the following CM the bank pays off  $d_h$  units of goods to each deposit holder and receives the payoff from its asset holdings.

A deposit contract is essentially a debt contract through which a financial intermediary borrows from depositors. Given limited commitment and no memory, the intermediary liabilities must be secured by its asset holdings. Unlike other financial intermediaries, however, Home banks are further constrained due to a capital requirement imposed by the Home government. The capital constraint (5) ensures that the value of the bank's liabilities cannot exceed a fraction  $1 - \delta$  of the risk-adjusted value of the bank's assets at maturity. Here, only a fraction  $1 - \theta$  of assets denominated in foreign currency is counted towards the risk-adjusted asset value.<sup>10</sup>

A Foreign bank's problem in equilibrium is analogous to that of the Home bank, except for the collateral constraint. A Foreign bank writes deposit contracts with bank depositors in the Foreign country (*Foreign bank depositors*, hereafter), who meet a type c or a type d seller in the following DM. Let  $(k_f, c_f, d_f)$  denote the terms of the Foreign bank's deposit contracts, where  $k_f$  is the quantity of goods deposited by each depositor in the CM,  $c_f$  is the quantity of Foreign currency the depositor can withdraw after the CM closes, and  $d_f$  is the quantity of claims to goods in the following CM for those not withdrawing currency. Also, let  $(m_f, b_f, b_f^*, e_f)$  denote the Foreign bank asset portfolio, where  $m_f$  is the quantity of Home reserves,  $b_f$  is the quantity of Home bonds,  $b_f^*$  is the quantity of Foreign bonds, and  $e_f$  is the quantity of interbank borrowing. Then, a Foreign bank's problem in equilibrium is given by:

$$\max_{k_f, c_f^*, d_f, m_f, b_f, b_f^*, e_f} \left[ -k_f + \rho u \left( \frac{\beta c_f^*}{\mu^*} \right) + (1 - \rho) u \left( \beta d_f \right) \right]$$
(7)

subject to

$$k_f + z_e e_f - \rho c_f^* - z_m m_f - z_b b_f - z_b^* b_f^* - \beta (1 - \rho) d_f + \frac{\beta (m_f + b_f - e_f)}{\mu} + \frac{\beta b_f^*}{\mu^*} \ge 0, \qquad (8)$$

$$-(1-\rho)d_f - \frac{e_f(1-I_f)}{\mu} + \frac{(1-\theta)(m_f + b_f - e_f I_f)}{\mu} + \frac{b_f^*}{\mu^*} \ge 0,$$
(9)

$$k_f, c_f^*, d_f, m_f, b_f, b_f^* \ge 0.$$
(10)

The Foreign bank is subject to a collateral constraint, instead of a capital constraint, implying that the Foreign bank is less constrained by the size of its balance sheet than the Home bank.<sup>11</sup> The collateral constraint, inequality (9), ensures that the Foreign bank's net payoff from repaying its liabilities is at least as large as the net payoff from defaulting.

Mutual funds write deposit contracts with buyers in both Home and Foreign countries. With probability  $\pi\gamma$ , each mutual fund depositor meets a type *b* seller who accepts only Home bonds in the DM, and with probability  $\pi(1 - \gamma)$ , they meet a type  $b^*$  seller who accepts only Foreign

<sup>&</sup>lt;sup>10</sup>In practice, other things equal, foreign currency-denominated assets are deemed riskier than domestic currency-denominated ones because the value of foreign currency-denominated assets is more volatile.

<sup>&</sup>lt;sup>11</sup>The collateral constraint could be considered as the capital constraint with  $\delta = 0$ .

bonds. With probability  $1 - \pi$ , each mutual fund depositor meets a type d seller and can use their claims to deposits for DM transactions. Considering the depositors' liquidity needs, mutual funds design a contract that permits a withdrawal of Home bonds and Foreign bonds after the CM closes. Let  $(k_g, \hat{b}_g, \hat{b}_g^*, d_g)$  denote the terms of deposit contracts offered by each mutual fund in equilibrium, where  $k_g$  is the quantity of goods deposited by each depositor in the CM,  $\hat{b}_g$  and  $\hat{b}_g^*$  are the quantities of Home bonds and Foreign bonds, respectively, the depositor can withdraw after the CM closes, and  $d_g$  is the quantity of claims to goods in the following CM for those not withdrawing currency. Also, let  $(b_g, b_g^*, e_g)$  denote the asset portfolio acquired by the mutual fund, where  $b_g$  is the quantity of Home bonds,  $b_g^*$  is the quantity of Foreign bonds, and  $e_g$  is the quantity of interbank borrowing.

Then, the mutual fund's problem in equilibrium is given by:

$$\max_{k_g, \hat{b}_g, \hat{b}_g^*, d_g, b_g, b_g^*, e_g} \left[ -k_g + \pi \gamma u \left( \frac{\beta \hat{b}_g}{\mu} \right) + \pi (1 - \gamma) u \left( \frac{\beta \hat{b}_g^*}{\mu^*} \right) + (1 - \pi) u \left( \beta d_g \right) \right]$$
(11)

subject to

$$k_g + z_e e_g - z_b b_g - z_b^* b_g^* - \beta (1 - \pi) d_g + \frac{\beta (b_g - \pi \gamma \hat{b}_g - e_g)}{\mu} + \frac{\beta [b_g^* - \pi (1 - \gamma) \hat{b}_g^*]}{\mu^*} \ge 0,$$
(12)

$$-(1-\pi)d_g - \frac{e_g}{\mu} + \frac{b_g - \pi\gamma\hat{b}_g}{\mu} + \frac{(1-\theta)[b_g^* - \pi(1-\gamma)\hat{b}_g^*]}{\mu^*} \ge 0,$$
(13)

$$k_g, d_g, b_g, b_g^* \ge 0, \quad 0 \le \hat{b}_g \le b_g, \quad 0 \le \hat{b}_g^* \le b_g^*.$$
 (14)

Similar to the Foreign bank's problem, the mutual fund is subject to a collateral constraint. We assume that mutual funds are based in the Home country and their balance sheets are denominated in the Home currency, so only a fraction  $1 - \theta$  of assets denominated in Foreign currency is counted towards the value of collateral.

In the model, Home banks face a higher balance sheet cost than Foreign banks and mutual funds due to the capital constraint. This assumption reflects our observation that traditional banks, such as U.S. depository institutions, must comply with tighter financial regulations than other financial institutions. Also, it reflects the differences in leverage ratio calculations across jurisdictions that make U.S. banks face a tighter balance sheet constraint than do the U.S. branches of foreign banks.

#### 3.2 Definition of equilibrium

We will define and characterize a stationary equilibrium where the collateral constraints matter for the behavior of financial intermediaries. In this model, collateral constraints bind when there is a shortage of collateral, and the supply of collateralizable assets is partly determined by fiscal policies, v and  $v^*$ . Therefore, we will assume that fiscal authorities exogenously set v and  $v^*$  to be sufficiently low, to focus on cases where collateral constraints bind in equilibrium.

We claim that, in equilibrium, the discounted net payoff for each intermediary must be zero. This occurs because otherwise there would be an opportunity for an intermediary to change its contract

in a way to attract more depositors and earn a higher payoff. This implies that, in equilibrium, constraints (4), (8), and (12) must hold with equality.

We let  $x_c$  and  $x_d$  denote the consumption quantities of Home bank depositors in DM meetings, respectively, with a type c seller and a type d seller. That is,  $x_c = \frac{\beta c_h}{\mu}$  and  $x_d = \beta d_h$ . Then, the first-order conditions for a Home bank's problem can be written as:

$$\mu = \beta [u'(x_c) - \delta u'(x_d) + \delta], \qquad (15)$$

$$z_b \ge \frac{\beta[(1-\delta)u'(x_d)+\delta]}{\mu},\tag{16}$$

$$z_b^* \ge \frac{\beta\{(1-\delta)[(1-\theta)u'(x_d) + \theta] + \delta\}}{\mu^*},\tag{17}$$

$$z_m \ge \frac{\beta[(1-\delta)u'(x_d) + \delta]}{\mu},\tag{18}$$

$$z_e \le \frac{\beta u'(x_d)}{\mu},\tag{19}$$

$$z_e \ge \frac{\beta[(1-\delta)u'(x_d) + \delta]}{\mu}.$$
(20)

Equation (15) indicates that the Home bank chooses to hold a positive quantity of Home currency. This occurs because there is always a demand for currency withdrawals from Home bank depositors. The bank may or may not hold government bonds and reserves. Each of equations (16), (17), and (18) holds with equality if the bank holds a positive quantity of the associated asset—Home bonds, Foreign bonds, and Home reserves, respectively. This gaurantees that the discounted net payoff from holding the asset is zero in equilibrium. In contrast, each equation holds with inequality if the bank chooses not to hold the asset. In other words, the bank does not hold the asset if the discounted net payoff from holding it is negative. Also, the bank may or may not trade on the interbank market. If the bank borrows on the interbank market,  $e_h > 0$  and equation (19) holds with equality, while if the bank lends,  $e_h < 0$  and equation (20) holds with equality.

Similarly, we can derive the first-order conditions for a Foreign bank's problem. Let  $x_c^*$  and  $x_d^*$  denote the consumption quantities of Foreign bank depositors in DM meetings, respectively, with a type c seller and a type d seller. That is,  $x_c^* = \frac{\beta c_f^*}{\mu^*}$  and  $x_d^* = \beta d_f$ . Then, the following conditions

must hold in equilibrium:

$$\mu^* = \beta u'\left(x_c^*\right),\tag{21}$$

$$z_b \ge \frac{\beta[(1-\theta)u'(x_d^*) + \theta]}{\mu},\tag{22}$$

$$z_b^* \ge \frac{\beta u'(x_d^*)}{\mu^*},\tag{23}$$

$$z_m \ge \frac{\beta[(1-\theta)u'(x_d^*) + \theta]}{\mu},\tag{24}$$

$$z_e \le \frac{\beta u'(x_d^*)}{\mu},\tag{25}$$

$$z_e \ge \frac{\beta[(1-\theta)u'(x_d^*) + \theta]}{\mu}.$$
(26)

Finally, let  $x_b$ ,  $x_b^*$ , and  $x_{gd}$  denote the consumption quantities of mutual fund depositors in DM meetings with a type *b* seller, a type  $b^*$  seller, and a type *d* seller. This implies that  $x_b = \frac{\beta \hat{b}_g}{\mu}$ ,  $x_b^* = \frac{\beta \hat{b}_g^*}{\mu^*}$ , and  $x_{gd} = \beta d_g$ . Then, the first-order conditions for a mutual fund's problem can be written as:

$$z_b = \frac{\beta u'(x_b)}{\mu},\tag{27}$$

$$u'(x_b) \ge u'(x_{gd}), \qquad if \ \pi \gamma x_b = \frac{\beta b_g}{\mu}$$
(28)

$$u'(x_b) = u'(x_{gd}), \qquad if \ \pi \gamma x_b < \frac{\beta b_g}{\mu}$$
(29)

$$z_b^* = \frac{\beta u'(x_b^*)}{\mu^*},$$
(30)

$$u'(x_b^*) \ge (1-\theta)u'(x_{gd}) + \theta, \qquad if \ \pi (1-\gamma)x_b^* = \frac{\beta b_g^*}{\mu^*}$$
(31)

$$u'(x_b^*) = (1 - \theta)u'(x_{gd}) + \theta, \qquad if \ \pi (1 - \gamma)x_b^* < \frac{\beta b_g^*}{\mu^*}$$
(32)

$$z_e = \frac{\beta u'(x_{gd})}{\mu}.$$
(33)

Equations (27)-(32) determine the quantities of bonds that can be withdrawn by the depositors after the CM closes, and whether the mutual fund holds more bonds than the required quantities for withdrawals. Equation (33) guarantees that the mutual fund is indifferent between borrowing and lending on the interbank market.

Asset market clearing conditions (the sum of the demands for each asset is equal to the corre-

sponding supply) are given by:

$$\alpha \rho c_h = \bar{c},\tag{34}$$

$$\alpha b_h + \alpha^* b_f + (2 - \alpha - \alpha^*) b_g = \overline{b}, \tag{35}$$

$$\alpha m_h + \alpha^* m_f = \bar{m},\tag{36}$$

$$\alpha^* \rho c_f^* = \bar{c}^*,\tag{37}$$

$$\alpha b_h^* + \alpha^* b_f^* + (2 - \alpha - \alpha^*) b_g^* = \bar{b}^*,$$
(38)

$$\alpha e_h + \alpha^* e_f + (2 - \alpha - \alpha^*) e_g = 0, \tag{39}$$

Equations (34)-(36) and (39) are associated with the markets for currency, government bonds, reserves, and interbank loans in the Home country, and equations (37) and (38) are related to the markets for currency and government bonds in the Foreign country.

As the law of one price holds in the CM, the nominal depreciation rate of the Home currency from the current period to the future period is determined by:

$$\frac{\xi_{+1}}{\xi} = \frac{\mu}{\mu^*}.$$
(40)

That is, the ratio of the Home inflation rate to the Foreign inflation rate determines the depreciation rate of the Home currency. From (27) and (30), the depreciation rate of the Home currency can also be expressed as:

$$\frac{\xi_{\pm1}}{\xi} = \frac{z_b^*}{z_b} \cdot \frac{u'(x_b)}{u'(x_b^*)}.$$
(41)

Equation (41) shows that the nominal interest rate differential, represented by  $z_b^*/z_b$ , is not the only determinant of the depreciation rate. We can show that the real interest rate on government bonds can differ across countries, implying a potential breakdown of uncovered interest rate parity. Government bonds can be used as liquidity-providing assets and collateral. Due to this non-pecuniary payoff, government bonds receive a liquidity premium, leading to a low real interest rate. So, depending on how they are used and the degree of liquidity needs, the real interest rate varies across different government bonds. Finally, we can define an equilibrium as follows.

**Definition** Given exogenous fiscal policies  $(v, v^*)$ , the Home central bank's monetary policy  $(z_m, \omega)$ , and the Foreign central bank's monetary policy  $z_b^*$ , a stationary equilibrium consists of the quantities of consumption in DM meetings  $(x_c, x_d, x_c^*, x_d^*, x_b, x_b^*, x_{gd})$ , the quantities of assets held by intermediaries  $(\{c_i, c_i^*, m_i, b_i, b_i^*, e_i\}_{i=h,f,g})$ , asset prices  $(z_b, z_e)$ , gross inflation rates  $(\mu, \mu^*)$ , and the nominal depreciation rate of Home currency  $\frac{\xi+1}{\xi}$ , satisfying (5), (9), and (13) with equality (capital and collateral constraints), (1)-(2) (fiscal policy rules), (15)-(33) (the first-order conditions for intermediaries' problems), (34)-(39) (asset market-clearing conditions), and (40).

**Proposition 1** Suppose v and  $v^*$  are sufficiently small such that

$$(1-\theta)v + v^* < x^e \left\{ (1-\rho) [\frac{\alpha(1-\theta)}{1-\delta} + \alpha^*] + (2-\alpha - \alpha^*) [1-\theta(1-\pi + \pi\gamma)] \right\},$$

where  $u'(x^e) = 1$ . Then, inequalities (5), (9), and (13) hold with equality,  $b_h^* = 0$ , and  $b_g^* = \pi (1-\gamma) \hat{b}_g^*$  in equilibrium.

Under the condition presented in Proposition 1, the capital constraint for Home banks, (5), and collateral constraints for Foreign banks and mutual funds, (9) and (13) respectively, bind in equilibrium. Moreover, Home banks do not hold Foreign bonds, i.e.,  $b_h^* = 0$ , favoring other assets to meet the capital requirement. Mutual funds hold some Foreign bonds but only to meet their depositors' withdrawal requests, i.e.,  $b_g^* = \pi (1 - \gamma) \hat{b}_g^*$ . In what follows, we will consider cases where v and  $v^*$  satisfy this condition.

### 4 Equilibrium with Inactive Interbank Market: A Baseline Case

In the previous section, we established a sufficient condition under which the balance sheet constraints of financial intermediaries—capital and collateral constraints—bind in equilibrium. In such cases, monetary policies of the two countries can affect the degree of collateral shortage in the financial market, thereby altering asset prices. In particular, the size of the Home central bank's balance sheet  $\omega$  directly affects the tightness of intermediary balance sheet constraints since an increase in  $\omega$  involves the central bank's swap of reserves for government bonds, affecting the composition of assets held by financial intermediaries.

If  $\omega$ , the size of the Home central bank's balance sheet, is sufficiently small, the quantity of government bonds outstanding is sufficiently large so that all financial intermediaries hold these bonds in their asset portfolios. However, as  $\omega$  increases, the quantity of reserves and currency issued by the Home central bank increases relative to Home bonds outstanding. Eventually, a rise in the price of Home bonds (or a fall in its nominal interest rate) will prevent banks from holding these assets, and therefore, their price become higher than that of reserves. If  $\omega$  becomes very large, mutual funds hold Home bonds only for providing liquidity services, and Home bonds are not used as collateral any longer.

**Proposition 2** If  $\delta > \theta$ , there exists a set of parameters leading to an active interbank market where Foreign banks borrow from mutual funds.

The above proposition states that a tight capital constraint, represented by a high  $\delta$ , prevents Home banks from participating in an interbank market, while Foreign banks may borrow from mutual funds. Specifically, when  $\omega$  (the size of the Home central bank's balance sheet) becomes sufficiently high, mutual funds begin to hold interbank loans as an alternative to Home bonds. Foreign banks begin to borrow at a lower interest rate in the interbank market to earn higher interest on reserves.

**Proposition 3** Suppose that  $v^*$  is sufficiently large such that

$$v^* > \alpha^* \rho x_c^* u'(x_c^*) + (2 - \alpha - \alpha^*)(1 - \gamma)\pi x_b^* u'(u_b^*)$$

in equilibrium. Then,  $b_f^* > 0$  and financial markets are interconnected in that changes in Foreign bond price influence other asset prices and vice versa.

Proposition 3 presents a condition under which financial markets are connected across countries. If the real value of consolidated Foreign government liabilities  $v^*$  is sufficiently small so that Foreign bonds are used only as a means of payment, the market for Foreign bonds will be completely separated from other asset markets. In this case, the price of Foreign bonds (or the interest rate on Foreign bonds) will be independent of other asset prices. However, if  $v^*$  is sufficiently large, Foreign bonds are used as collateral as well as a means of payment. In this case, changes in the price of Foreign bonds are accompanied by changes in the prices of other collateralizable assets. In what follows, we will consider cases where  $v^*$  is sufficiently large to satisfy the condition in Proposition 3.

#### 4.1 Case 0: Home bonds are held by both banks and mutual funds

If the quantity of Home bonds is sufficiently large relative to central bank liabilities (i.e., currency and reserves), there does not arise an interest rate gap between Home bonds and reserves. In this scenario, there are no benefits for banks from engaging in arbitrage and no need of alternative interest-bearing assets for mutual funds. Therefore, the interbank market becomes inactive.

In this section, we will examine equilibria where Home bonds are sufficiently plentiful but not too plentiful so that intermediary balance sheet constraints bind. Due to a sufficiently large supply of Home bonds, Home banks, Foreign banks, and mutual funds all hold Home bonds in their asset portfolios. Suppose that  $\delta$ ,  $\theta$ , v, and  $v^*$  satisfy the conditions presented in Propositions 1-3. Then, we can derive the following equilibrium conditions, noting that the first-order conditions (18), (23), (24), and (25) hold with equality, and that  $\pi \gamma \hat{b}_g < b_g$  and  $\pi (1 - \gamma) \hat{b}_g^* = b_g^*$ .

$$(1-\delta)u'(x_d) + \delta = (1-\theta)u'(x_d^*) + \theta,$$
(42)

$$z_m = \frac{(1-\delta)u'(x_d) + \delta}{u'(x_c) - \delta u'(x_d) + \delta},\tag{43}$$

$$z_b^* = \frac{u'(x_b^*)}{u'(x_c^*)},\tag{44}$$

$$x_b^* = x_d^*,\tag{45}$$

$$u'(x_b) = (1-\delta)u'(x_d) + \delta.$$
(46)

Equation (42) implies that the consumption quantities in DM meetings with a type-d seller are

positively correlated across the countries. Equations (43) and (44) show how monetary policy affects the DM consumption quantities in the two countries. Essentially, increasing the target price of an asset (or equivalently, lowering the nominal interest rate target) increases the consumption quantity in DM meetings involving currency relative to other DM meetings. The structure of asset prices can be expressed as:

$$\frac{\beta[(1-\delta)u'(x_d)+\delta]}{\mu} = z_m = z_e = \frac{\beta u'(x_{gd})}{\mu} = \frac{\beta[(1-\theta)u'(x_d^*)+\theta]}{\mu} = \frac{\beta u'(x_b)}{\mu} = z_b.$$
 (47)

Since the prices of reserves, interbank loans, and Home bonds are identical, Home and Foreign banks are indifferent between reserves and Home bonds. Mutual funds are also indifferent between lending on the interbank market and acquiring Home bonds because both assets serve as collateral at the margin. Consequently, the interbank market becomes inactive due to the lack of demand for interbank loans.

Given the condition in Proposition 3 ensuring endogenously interconnected asset markets, all financial intermediaries are subject to a consolidated collateral constraint within the global financial system. From (1), (2), (5), (9), (13), and (34)-(47), noting that  $b_h \ge 0$ ,  $b_f \ge 0$ ,  $b_h^* = e_h = e_f = e_g = 0$ , the consolidated collateral constraint can be written as:

$$\alpha \rho x_c \left[ u'(x_c) + \frac{\delta}{1-\delta} \right] + \alpha (1-\rho) x_d \left[ u'(x_d) + \frac{\delta}{1-\delta} \right]$$
$$+ \alpha^* \rho x_c^* u'(x_c^*) \left[ 1 + \frac{\theta}{(1-\theta)u'(x_d^*)} \right] + \alpha^* (1-\rho) x_d^* \left[ u'(x_d^*) + \frac{\theta}{1-\theta} \right]$$
$$+ (2-\alpha-\alpha^*) \left\{ (1-\pi+\pi\gamma) x_b u'(x_b) + \pi (1-\gamma) x_b^* \left[ u'(x_b^*) + \frac{\theta}{1-\theta} \right] \right\}$$
$$= v + \left[ 1 + \frac{\theta}{(1-\theta)u'(x_d^*)} \right] v^*. \tag{48}$$

Equation (48) states that the aggregate demand for collateral (represented by the left-hand side) must be equal to its aggregate supply (the right-hand side) in equilibrium. The quantities of collateral demanded and supplied are adjusted to reflect the balance sheet costs faced by intermediaries.

Using equations (42)-(48), we can solve for the consumption quantities in the DM meetings  $(x_c, x_d, x_c^*, x_d^*, x_b, x_b^*, x_{gd})$  given monetary and fiscal policies  $(z_m, \omega, z_b^*, v, v^*)$ . Then, equations (15) and (21) solve for the Home and Foreign inflation rates  $\mu$  and  $\mu^*$ , and equation (47) determines the prices of Home bonds and interbank loans  $z_b$  and  $z_e$ . Finally, equation (40) determines the nominal depreciation rate of the Home currency  $\frac{\xi_{\pm 1}}{\xi}$ .

For this type of equilibrium to exist, the equilibrium solution must satisfy:

$$0 \le \omega \le v - (2 - \alpha - \alpha^*) \left\{ (1 - \pi) x_d^* u'(x_d^*) + \pi \gamma x_b u'(x_b) \right\},\tag{49}$$

According to the condition (49), the size of the Home central bank's balance sheet  $\omega$  must be sufficiently small to the extent that a sufficient amount of Home bonds circulated in the financial

$x_c$	$x_d$	$x_c^*$	$x_d^*$	$x_b$	$x_b^*$	$x_{gd}$	$\mu$	$\mu^*$
$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$z_m$	$z_e$	$z_b$	$z_e - z_m$		$z_b - z_e$		$z_b^*$	$\frac{\xi_{\pm 1}}{\xi}$
$\uparrow$	$\uparrow$	$\uparrow$	•		•			$\downarrow$

Table 2: Effects of an increase in  $z_m$ 

system allow all financial intermediaries to hold Home bonds.

Now, we can analyze the effects of the Home central bank's monetary policy on the Home country and its transmission to the Foreign country. Since the central bank's balance sheet policy, which involves changes in  $\omega$ , is irrelevent in this case, we will focus on the effects of a conventional interest rate policy.

#### 4.1.1 Conventional easing: a fall in the nominal interest rate on reserves

Suppose that the Home central bank decreases the nominal interest rate on reserves, i.e., increases  $z_m$ , while maintaining the size of its balance sheet  $\omega$  at the current level. From (43), an increase in  $z_m$  leads to an increase in  $x_c$  and a decrease in  $x_d$ . This indicates a substitution of Home currency for reserves, increasing consumption in DM meetings with type c sellers and decreasing consumption in those with type d sellers. From (42), (45), (46), and (47), a decrease in  $x_d$  is accompanied by decreases in  $x_d^*$ ,  $x_{gd}$ ,  $x_b$ , and  $x_b^*$ . This occurs because a higher price of reserves leads to a higher demand for government bonds, driving up their prices and reducing real interest rates. The resulting reduction in the redemption value of these assets effectively decreases the stock of collateral in the financial system. As a result, the consumption quantities in DM meetings involving government bonds ( $x_b, x_b^*$ ) and the consumption quantities in those supported by collateral-backed deposits ( $x_d, x_d^*, x_{gd}$ ) all decrease. The price of Foreign bonds  $z_b^*$  tends to increase, but the Foreign central bank maintains it at the target level by selling Foreign bonds in the market. This leads to a decrease in the quantity of Foreign currency outstanding, so from (44),  $x_c^*$  decreases.

From (15), the Home inflation rate  $\mu$  falls, and from (47), the prices of Home bonds and interbank loans,  $z_b$  and  $z_e$ , both rise. An increase in  $z_m$  also affects the Foreign inflation rate  $\mu^*$  and the nominal exchange rate. From (21), the Foreign inflation rate  $\mu^*$  rises, so from (40)-(41), the depreciation rate of the Home currency  $\frac{\xi_{\pm 1}}{\xi}$  falls. Specifically, a decrease in the nominal interest rate on Home bonds decreases the demand for Home bonds relative to Foreign bonds. This causes a current depreciation of the Home currency, i.e., a rise in the relative price of Foreign currency  $\xi$ , leading to a fall in the depreciation rate  $\frac{\xi_{\pm 1}}{\xi}$ . These results are summarized in Table 2.

### 5 Equilibrium with Active Interbank Market

In this section, we will focus on cases where the nominal interest rate on Home bonds falls below the interest rate on bank reserves. This scenario arises when the central bank significantly expands its balance sheet by exchanging reserves for Home bonds. This intervention increases the quantity of reserves, which can only be held by banks, while reducing the quantity of Home bonds, which can be held by any financial intermediaries. Thus, this intervention acts to raise the liquidity premium on Home bonds relative to reserves, causing the nominal interest rate on Home bonds to fall below the interest rate on reserves.

When there is a nominal interest rate gap between reserves and government bonds, financial intermediaries create another type of asset through the interbank market: interbank loans. Interbank loans essentially enable mutual funds to hold reserves indirectly. A necessary condition for the nominal interest rate gap to emerge is a sufficient scarcity of collateralizable assets. In other words, the interbank market becomes active when collateral is sufficiently scarce. There are four different scenarios where Foreign banks borrow from mutual funds on the interbank market. Assuming the condition in Proposition 3, we will examine the two cases where asset markets are endogenously interconnected across the countries.<sup>12</sup>

#### 5.1 Case 1: Home bonds are used as collateral at the margin

If Home bonds are sufficiently scarce but not too scarce, they are held exclusively by mutual funds, which use these assets to back deposits at the margin. In this case, Home banks receive deposits and hold only reserves, while Foreign banks receive deposits and borrow on the interbank market to hold reserves and Foreign bonds. Mutual funds receive deposits and acquire Home bonds, Foreign bonds, and interbank loans. Noting that the first-order conditions (18), (23), (24), and (25) hold with equality, and that  $\pi \gamma \hat{b}_g < b_g$  and  $\pi (1 - \gamma) \hat{b}_g^* = b_g^*$ , we can derive the following four conditions same as those presented in Case 0.

$$(1-\delta)u'(x_d) + \delta = (1-\theta)u'(x_d^*) + \theta, \qquad (42)$$

$$z_m = \frac{(1-\delta)u'(x_d) + \delta}{u'(x_c) - \delta u'(x_d) + \delta},\tag{43}$$

$$z_b^* = \frac{u'(x_b^*)}{u'(x_c^*)},\tag{44}$$

$$x_b^* = x_d^*. \tag{45}$$

The structure of asset prices can be expressed as:

$$\frac{\beta[(1-\delta)u'(x_d)+\delta]}{\mu} = z_m < z_e = \frac{\beta u'(x_d^*)}{\mu} = \frac{\beta u'(x_{gd})}{\mu} = \frac{\beta u'(x_b)}{\mu} = z_b.$$
 (50)

This structure shows that Home and Foreign banks strictly prefer reserves to Home bonds. It also indicates that banks may engage in arbitrage by borrowing on the interbank market and acquiring reserves. However, Home banks do not borrow on the interbank market as they are heavily constrained by capital requirements, which imply a high balance sheet cost. Foreign banks

<sup>&</sup>lt;sup>12</sup>In the other two cases, asset markets are segmented in the sense that different financial intermediaries hold entirely distinct asset portfolios. Consequently, changes in the price of an asset held by one type of intermediaries do not affect the prices of assets held by other types of intermediaries.

actively borrow on the interbank market to earn arbitrage profits, but they also face a balance sheet constraint due to a shortage of collateral. Therefore, an interest rate gap between reserves and interbank loans remains in equilibrium. Mutual funds are indifferent between lending on the interbank market and acquiring Home bonds because both assets serve as collateral at the margin.

Similar to Case 0, all financial intermediaries are subject to a consolidated collateral constraint due to the interconnectness of asset markets across countries. From (1), (2), (5), (9), (13), (34)-(45), and (50), noting that  $b_h = b_f = b_h^* = e_h = 0$ , the consolidated collateral constraint can be written as:

$$\alpha(1-\rho)x_{d}\left[u'(x_{d})+\frac{\delta}{1-\delta}\right] + \alpha\rho x_{c}\left[u'(x_{c})+\frac{\delta}{1-\delta}\right]$$

$$+ [\alpha^{*}(1-\rho)+(2-\alpha-\alpha^{*})(1-\pi)]x_{d}^{*}\left[u'(x_{d}^{*})+\frac{\theta}{1-\theta}\right]$$

$$+ (2-\alpha-\alpha^{*})\pi\left\{\gamma x_{b}\left[u'(x_{b})+\frac{\theta}{1-\theta}\right] + (1-\gamma)x_{b}^{*}\left[u'(x_{b}^{*})+\frac{\theta}{1-\theta}\right]\right\}$$

$$+ \alpha^{*}\rho x_{c}^{*}u'(x_{c}^{*})\left[1+\frac{\theta}{(1-\theta)u'(x_{d}^{*})}\right]$$

$$= (v+v^{*})\left[1+\frac{\theta}{(1-\theta)u'(x_{d}^{*})}\right] - \frac{\theta\omega}{(1-\theta)u'(x_{d}^{*})}.$$

$$(51)$$

Equation (51) ensures that the aggregate demand for collateral is equal to its aggregate supply in equilibrium. The quantities demanded and supplied are adjusted to reflect the balance sheet costs faced by intermediaries.

Using equations (42)-(45) and (50)-(51), we can solve for the consumption quantities in the DM meetings  $(x_c, x_d, x_c^*, x_d^*, x_b, x_b^*, x_{gd})$  given monetary and fiscal policies  $(z_m, \omega, z_b^*, v, v^*)$ . Then, equations (15) and (21) solve for the Home and Foreign inflation rates  $\mu$  and  $\mu^*$ , and equation (50) determines the prices of Home bonds and interbank loans  $z_b$  and  $z_e$ . Finally, equation (40) determines the nominal depreciation rate of the Home currency  $\frac{\xi_{\pm 1}}{\xi}$ .

A necessary condition for the existence of this equilibrium is given by:

$$v - (2 - \alpha - \alpha^*) \left\{ (1 - \pi) x_d^* u'(x_d^*) + \pi \gamma x_b u'(x_b) \right\} \le \omega \le v - (2 - \alpha - \alpha^*) \pi \gamma x_b u'(x_b),$$
(52)

The condition (52) states that the size of the Home central bank's balance sheet  $\omega$  must be sufficiently large so that an insufficient amount of collateral in the financial system induces intermediaries to trade interbank loans. Also,  $\omega$  must be sufficiently small to ensure a sufficient quantity of Home bonds in the financial system that can be used by mutual fund depositors in the DM meetings with type *b* sellers.

Now, we can analyze the effects of the Home central bank's monetary policy on the Home country and its transmission to the Foreign country.

$x_c$	$x_d$	$x_c^*$	$x_d^*$	$x_b$	$x_b^*$	$x_{gd}$	$\mu$	$\mu^*$
$\uparrow$	$\downarrow$	$\uparrow$						
$z_m$	$z_e$	$z_b$	$z_e - z_m$		$z_b$ -	$-z_e$	$z_b^*$	$\frac{\xi_{\pm 1}}{\xi}$
$\uparrow$	1	1	1		•			$\downarrow$

Table 3: Effects of an increase in  $z_m$ 

#### 5.1.1 Conventional easing: a fall in the nominal interest rate on reserves

Suppose that the Home central bank lowers the nominal interest rate on reserves (i.e., raises  $z_m$ ) without changing the size of its balance sheet  $\omega$ . The qualitative effects of this intervention on the DM consumption quantities and asset prices are the same as those in Case 0. That is, from (42)-(45) and (50), the consumption quantities in all DM meetings supported by government bonds  $(x_b, x_b^*)$ , collateral-backed deposits  $(x_d, x_d^*, x_{gd})$ , and Foreign currency  $x_c^*$  decrease, while the consumption quantity in DM meetings involving Home currency  $x_c$  increases. Also, from (15), (21), and (40)-(41), the Home inflation rate  $\mu$  falls while the Foreign inflation rate  $\mu^*$  rises, resulting in a fall in the depreciation rate of the Home currency  $\frac{\xi+1}{\xi}$ . Finally, from (50), the prices of Home bonds and interbank loans,  $z_b$  and  $z_e$ , both rise.

What differs from the previous case is that in this equilibrium there is a price gap between reserves and interbank loans, and monetary policy can affect this price gap. Using (41), (42), and (44), the difference in price between interbank loans and reserves,  $z_e - z_m$ , can be written as:

$$z_e - z_m = \frac{\beta \theta \left[ u'(x_d^*) - 1 \right]}{\mu}.$$
(53)

Thus, the price difference, or nominal interest rate gap, increases as  $z_m$  increases. These results are summarized in Table 3.

#### 5.1.2 Unconventional easing: a balance sheet expansion

Suppose instead that the Home central bank increases the size of its balance sheet  $\omega$  while holding the nominal interest rate on reserves constant. Equation (51) shows that an increase in  $\omega$  effectively decreases the supply of collateralizable assets. Then, from (42)-(45) and (50)-(51), all DM consumption quantities  $(x_c, x_d, x_c^*, x_d^*, x_b, x_b^*, x_{gd})$  decrease. The Home central bank increases the size of its balance sheet by exchanging reserves for Home bonds. Since mutual funds cannot hold reserves, this intervention acts to tighten their collateral constraints, ultimately tightening the consolidated collateral constraint in the financial system. As a result, consumption decreases in all DM meetings involving deposits backed by collateral, i.e.,  $x_d, x_d^*$ , and  $x_{gd}$  decrease. The tightened collateral constraint also reduces the quantity of government bonds that can be used to provide liquidity services, thereby decreasing consumption in those DM meetings  $(x_b \text{ and } x_b^*)$ .

A decrease in the supply of collateralizable assets increases their prices, thereby reducing real interest rates. However, the nominal interest rates on assets held by Home banks remain constant

$x_c$	$x_d$	$x_c^*$	$x_d^*$	$x_b$	$x_b^*$	$x_{gd}$	$\mu$	$\mu^*$
$\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$
$z_m$	$z_e$	$z_b$	$z_e - z_m$		$z_b - z_e$		$z_b^*$	$\frac{\xi_{\pm 1}}{\xi}$
•	$\uparrow$	$\uparrow$	$\uparrow$		•			$\downarrow$

Table 4: Effects of an increase in  $\omega$ 

because the nominal interest rate on reserves is administratively set by the Home central bank, and Home currency pays zero nominal interest. This implies that a decrease in real interest rates must be accompanied by a rise in the Home inflation rate  $\mu$  in equilibrium. Consequently, a fall in real interest rates only tightens the balance sheet constraints of Home banks without altering the price of reserves relative to Home currency. Therefore, Home banks reduce their holdings of Home currency and reserves as if there is a negative wealth effect. The price of Foreign bonds  $z_b^*$  also tends to rise, causing the Foreign central bank to exchange Foreign bonds for currency (an open-market sale) to maintain the price of Foreign bonds at the target level. This intervention mitigates the decrease in the supply of collateral at the expense of a decrease in Foreign currency outstanding. As the real quantities of Home and Foreign currencies outstanding decrease, consumption in currency-involved DM meetings ( $x_c$  and  $x_c^*$ ) decreases in both countries.

Using (15), (42)-(45), and (50), the price of interbank loans can also be expressed as:

$$z_e = \frac{z_m \left[ (1 - \delta) u'(x_d) + \delta - \theta \right]}{(1 - \theta) \left[ (1 - \delta) u'(x_d) + \delta \right]},$$
(54)

which equals the price of Home bonds, i.e.,  $z_e = z_b$ , from (50). We can show that  $z_e$  and  $z_b$  rise as  $x_d$  decreases. This occurs because a decrease in the effective stock of collateral leads to an increase in the prices of Home bonds and interbank loans. So, the difference in price between these assets and reserves,  $z_b - z_m$ , increases in equilibrium. Also, from (15) and (21), both inflation rates  $\mu$  and  $\mu^*$  rise as  $x_c$  and  $x_c^*$  decrease. Finally, from (41) and (50), the nominal depreciation rate of Home currency can be expressed as:

$$\frac{\xi_{\pm 1}}{\xi} = \frac{z_b^*}{z_b}.$$
(55)

According to equation (55), uncovered interest rate parity strictly holds in this case because the nominal depreciation rate only reflects the nominal interest rate gap between Home and Foreign bonds. Because  $z_b$  rises while  $z_b^*$  is held constant, the depreciation rate falls, implying a current depreciation of the Home currency followed by an expected appreciation in the next period.

Interestingly, the welfare of Foreign bank depositors decreases as their consumption quantities in DM meetings  $x_c^*$  and  $x_d^*$  both decrease. This occurs despite a rise in the nominal interest rate gap between interbank borrowings and reserves, leading to increased arbitrage profits for Foreign banks. Foreign banks utilize these increased arbitrage profits to increase DM consumption for their depositors, but the increase in consumption is outweighed by its decrease due to the tightened collateral constraint. Therefore,  $x_c^*$  and  $x_d^*$  can decrease even when Foreign banks earn more arbitrage profits. These results are summarized in Table 4.

#### 5.2 Case 2: Home bonds are used as medium of exchange at the margin

In equilibrium, the price mechanism serves to allocate each asset to economic activities where it can be used most efficiently. When Home bonds become extremely scarce, they are allocated only to certain DM meetings to be used as a medium of exchange. The composition of assets and liabilities of each financial intermediary remains the same as in the previous case. Home banks hold only reserves as assets and only deposits as liabilities. Foreign banks hold reserves and Foreign bonds as assets, and have deposits and interbank borrowings as liabilities. Mutual funds hold Home bonds, Foreign bonds, and interbank loans as assets, and have only deposits as liabilities. However, unlike the previous case, mutual funds do not use Home bonds to back their deposit liabilities. Instead, all Home bonds held by mutual funds are withdrawn by their depositors for DM transactions.

Noting that the first-order conditions (18), (23), (24), and (25) hold with equality, and that  $\pi \gamma \hat{b}_g = b_g$  and  $\pi (1 - \gamma) \hat{b}_g^* = b_g^*$ , we can derive some equilibrium conditions identical to those in the previous cases, such as equations (42)-(45). However, the price of Home bonds is higher than that of interbank loans because

$$\frac{\beta[(1-\delta)u'(x_d)+\delta]}{\mu} = z_m < z_e = \frac{\beta u'(x_d^*)}{\mu} = \frac{\beta u'(x_{gd})}{\mu} \le \frac{\beta u'(x_b)}{\mu} = z_b.$$
(56)

Similar to the previous cases, the global financial system as a whole faces a consolidated collateral constraint. From (1), (2), (5), (9), (13), (34)-(45), and (56), noting that  $b_h = b_f = b_h^* = e_h = 0$ , the consolidated collateral constraint can be expressed as:

$$\alpha(1-\rho)x_d \left[u'(x_d) + \frac{\delta}{1-\delta}\right] + \alpha\rho x_c \left[u'(x_c) + \frac{\delta}{1-\delta}\right]$$
$$+ \left[\alpha^*(1-\rho) + (2-\alpha-\alpha^*)(1-\pi)\right]x_d^* \left[u'(x_d^*) + \frac{\theta}{1-\theta}\right]$$
$$+ (2-\alpha-\alpha^*)\pi(1-\gamma)x_b^* \left[u'(x_b^*) + \frac{\theta}{1-\theta}\right]$$
$$+ \alpha^*\rho x_c^* u'(x_c^*) \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right]$$
$$= \omega + v^* \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right]. \tag{57}$$

In this equilibrium, there is another constraint relevant to mutual funds and DM meetings involving Home bonds, which is expressed as:

$$(2 - \alpha - \alpha^*)\pi\gamma x_b u'(x_b) = v - \omega.$$
(58)

Equation (58) states that the quantity for Home bonds demanded as a medium of exchange equals

$x_c$	$x_d$	$x_c^*$	$x_d^*$	$x_b$	$x_b^*$	$x_{gd}$	$\mu$	$\mu^*$
$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	•	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
$z_m$	$z_e$	$z_b$	$z_e - z_m$		$z_b - z_e$		$z_b^*$	$\frac{\xi_{\pm 1}}{\xi}$
$\uparrow$	$\uparrow$	$\uparrow$	1		?			$\downarrow$

Table 5: Effects of an increase in  $z_m$ 

the quantity supplied in the financial system. Since Home bonds are not part of the collateral in this equilibrium, a separate condition determines the quantity of consumption in DM meetings involving Home bonds  $x_b$ .

Equation (58) can be used to solve for  $x_b$ , and equations (42)-(45) and (56)-(57) can be used to solve for all other DM consumption quantities  $(x_c, x_d, x_c^*, x_d^*, x_b^*, x_{gd})$  given monetary and fiscal policies  $(z_m, \omega, z_b^*, v, v^*)$ . Then, equations (15) and (21) solve for the Home and Foreign inflation rates  $\mu$  and  $\mu^*$ , and equation (56) determines the prices of Home bonds and interbank loans  $z_b$  and  $z_e$ . Finally, equation (40) determines the nominal depreciation rate of the Home currency  $\frac{\xi+1}{\xi}$ .

For this type of equilibrium to exist, it is necessary for the equilibrium solution to satisfy the following conditions:

$$v - (2 - \alpha - \alpha^*)\pi\gamma x_d^* u'(x_d^*) \le \omega \le v, \tag{59}$$

The condition (59) states that the size of the Home central bank's balance sheet  $\omega$  must be very large to the extent that Home bonds are only used in DM meetings due to a severe shortage of Home bonds in the financial system.

#### 5.2.1 Conventional easing: a fall in the nominal interest rate on reserves

Suppose that the Home central bank decreases the nominal interest rate on reserves (or equivalently, increases  $z_m$ ) while holding  $\omega$ , the size of its balance sheet, constant. The effects of this intervention on the DM consumption quantities are qualitatively identical to those in the previous case, except that from (58)  $x_b$  is unaffected. From (43), an increase in  $z_m$  leads to an increase in  $x_c$  and a decrease in  $x_d$  because it causes a substitution of Home currency for reserves. Also, an increase in  $z_m$  reduces the effective stock of collateral in the financial system, tightening the consolidated collateral constraint as it does in the previous case. So, from (42), (44), (45), and (56), consumption quantities in DM meetings involving collateral-backed deposits  $(x_d, x_d^*, x_{gd})$  decrease. There is upward pressure on the price of Foreign bonds, prompting the Foreign central bank to swap Foreign bonds for currency to meet its policy target  $z_b^*$ . As a result, the quantity of Foreign currency outstanding decreases, and the quantity of consumption in Foreign currency-involved DM meetings  $x_c^*$  decreases. Consumption in Foreign bonds-involved DM meetings  $x_b^*$  also decreases because the Foreign central bank's intervention acts to reduce the real interest rate on Foreign bonds.

Similar to what happens in the previous case, from (15) and (21), the Home inflation rate  $\mu$ falls, while the Foreign inflation rate  $\mu^*$  rises. So, from (40), the depreciation rate of the Home

$x_c$	$x_d$	$x_c^*$	$x_d^*$	$x_b$	$x_b^*$	$x_{gd}$	$\mu$	$\mu^*$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\rightarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$z_m$	$z_e$	$z_b$	$z_e - z_m$		$z_b - z_e$		$z_b^*$	$\frac{\xi_{\pm 1}}{\xi}$
•	$\rightarrow$	$\uparrow$	$\downarrow$		$\uparrow$			?

Table 6: Effects of an increase in  $\omega$ 

currency  $\frac{\xi_{\pm 1}}{\xi}$  falls. This implies that the Home currency depreciates in the current period due to a lower nominal interest rate on Home bonds relative to Foreign bonds and is expected to appreciate in the next period. From (56), the prices of Home bonds and interbank loans,  $z_b$  and  $z_e$ , both rise, but the difference in price between Home bonds and interbank loans,  $z_b - z_e$ , is ambiguous. This is because a lower inflation rate tends to increase the difference, while a larger increase in the price of interbank loans decreases the difference. Finally, from (53) the price difference between interbank loans and reserves,  $z_e - z_m$ , increases. These results are summarized in Table 5.

#### 5.2.2 Unconventional easing: a balance sheet expansion

Suppose instead that the Home central bank expands its balance sheet, i.e., increases  $\omega$ , while holding the nominal interest rate on reserves constant. An increase in  $\omega$  increases the quantity of reserves and decreases the quantity of Home bonds in the financial system. From (57), this intervention increases the effective stock of collateral and relaxes the collateral constraint faced by intermediaries. This result contrasts with the previous case where an increase in  $\omega$  tightens the collateral constraint. In the previous case, both Home bonds and reserves are used as collateral at the margin. If the Home central bank swaps reserves for Home bonds, the effective stock of collateral decreases because reserves are a poor substitute for Home bonds as collateral. However, in the case considered here, Home bonds are only used to support a certain type of DM transaction. Thus, the Home central bank's asset swap acts to increase the quantity of collateralizable assets (reserves) and decrease the quantity of means of payment (Home bonds) in some DM transactions. Specifically, from (58), consumption in Home bonds-involved DM meetings  $x_b$  decreases, while from (42)-(45), (56), and (57) all other DM consumption quantities  $x_c, x_d, x_c^*, x_d^*, x_b^*$ , and  $x_{gd}$  increase.

An increase in the effective stock of collateralizable assets reduces their prices and relaxes the balance sheet constraint of Home banks. This works as a positive wealth effect for Home banks, increasing their holdings of Home currency and reserves. In equilibrium, an increase in the real quantity of their Home currency holdings (and the resulting increase in  $x_c$ ) is accompanied by a fall in the Home inflation rate  $\mu$ . Initially, there is downward pressure on the price of Foreign bonds  $z_b^*$ , which makes the Foreign central bank conduct an open-market purchase of Foreign bonds to maintain  $z_b^*$  at the target level. As a result, the quantity of Foreign currency outstanding in real terms increases, leading to an increase in consumption in DM meetings involving Foreign currency  $x_c^*$  and a fall in the Foreign inflation rate  $\mu^*$ .

From (54), the price of interbank loans  $z_e$  falls as  $x_d$  increases, implying that the price difference between interbank loans and reserves  $z_e - z_m$  decreases. In contrast, from (56), the price of Home bonds rises, causing the difference in price between Home bonds and interbank loans  $z_b - z_e$  to increase. The reason is that the market for Home bonds is essentially segmented from the one for reserves and other collaterlizable assets in this equilibrium. The Home central bank's balance sheet expansion involves an asset swap, which increases reserves and decreases Home bonds held by financial intermediaries. As interbank loans provide an alternative way to hold reserves, the price of interbank loans  $z_e$  falls as the quantity of reserves increases, while the price of Home bonds  $z_b$ rises.

As the inflation rate falls in both countries, the effect on the nominal depreciation rate of the Home currency, as indicated by (40), is ambiguous. Specifically, from (41), the nominal interest rate on Home bonds decreases (or  $z_b$  rises) relative to that of Foreign bonds, which tends to depreciate the Home currency in the current period. However, the real interest rate on Home bonds also relatively decreases, which appreciates the Home currency in nominal terms, making the effect on the nominal depreciation rate uncertain. This result demonstrates that uncovered interest rate parity does not necessarily hold in this case, as a fall in the nominal interest rate on Home bonds relative to Foreign bonds does not guarantee a nominal depreciation of the Home currency.

In contrast to the previous case where the Home central bank's balance sheet expansion reduces the welfare of Foreign bank depositors, their welfare increases in this case. The quantities of DM consumption for Foreign bank depositors  $x_c^*$  and  $x_d^*$  increase because Foreign banks benefit from the increase in the stock of collateral in the financial system. Even though the nominal interest rate gap between interbank borrowings and reserves falls, the relaxation of the consolidated collateral constraint allows Foreign banks to increase their holdings of Foreign currency and deposit claims. This, in turn, increases the corresponding consumption quantities in the DM meetings, thereby enhancing the welfare of Foreign bank depositors. These results are summarized in Table 6.

# 6 Overnight Reverse Repurchase Agreement Facility

Due to insufficient arbitrage, the gap between the interest on reserve balances (IORB) rate and the fed funds rate persisted. In response, the Federal Reserve announced in September 2014 that it would use an overnight reverse repurchase agreement (ON RRP) facility as a supplementary policy tool to better control the fed funds rate. This facility allows eligible counterparties to hold central bank liabilities overnight and earn interest—similar to reserve balances for depository institutions—but extends access to a wider range of entities such as banks, government-sponsored enterprises (GSEs), and mutual funds.

As long as the Fed supplies a sufficiently large amount of ON RRPs, their interest rate would serve as a lower bound for the fed funds rate. If the fed funds rate were to fall below the ON RRP rate, GSEs would prefer using the ON RRP facility over lending in the fed funds market. Therefore, the introduction of the ON RRP facility helps raise the federal funds rate and reduces arbitrage profits for foreign banks that rely on borrowing from GSEs in the fed funds market. In fact, as shown in Figure 2, foreign banks' share of total reserves and their reserves-to-total-assets ratio both significantly decreased between 2014 and 2019.<sup>13</sup> In this section, we will explore how the ON RRP facility affects domestic and global financial markets.

We extend the model by allowing the Home central bank to issue interest-bearing ON RRPs that can be held by mutual funds. Similar to reserves and interbank loans, these ON RRPs can be used as collateral to back intermediaries' deposit liabilities but cannot serve as a medium of exchange for type *b* sellers. Each unit of ON RRPs sells at price  $z_o$  in the CM and represents a claim to one unit of Home currency in the next CM. Let  $\omega$  denote the value of the Home central bank's liabilities (or the size of its balance sheet), which now consist of currency, reserves, and ON RRPs. Additionally, let  $\bar{o}$  denote the real quantity of ON RRPs held by the public, and  $\omega_o = z_o \bar{o}$  denote the value of ON RRPs outstanding, so that  $\omega = \bar{c} + z_m \bar{m} + \omega_o$ . We will analyze the last two cases from the previous section, where foreign banks actively borrow from mutual funds in the interbank market.

#### 6.1 Case 1: Home bonds are used as collateral at the margin

In the presence of an ON RRP facility, the consolidated collateral constraint (51) can be rewritten as:

$$\alpha(1-\rho)x_{d}\left[u'(x_{d})+\frac{\delta}{1-\delta}\right] + \alpha\rho x_{c}\left[u'(x_{c})+\frac{\delta}{1-\delta}\right]$$

$$+ \left[\alpha^{*}(1-\rho)+(2-\alpha-\alpha^{*})(1-\pi)\right]x_{d}^{*}\left[u'(x_{d}^{*})+\frac{\theta}{1-\theta}\right]$$

$$+ (2-\alpha-\alpha^{*})\pi\left\{\gamma x_{b}\left[u'(x_{b})+\frac{\theta}{1-\theta}\right] + (1-\gamma)x_{b}^{*}\left[u'(x_{b}^{*})+\frac{\theta}{1-\theta}\right]\right\}$$

$$+ \alpha^{*}\rho x_{c}^{*}u'(x_{c}^{*})\left[1+\frac{\theta}{(1-\theta)u'(x_{d}^{*})}\right]$$

$$= (v+v^{*})\left[1+\frac{\theta}{(1-\theta)u'(x_{d}^{*})}\right] - \frac{\theta\omega}{(1-\theta)u'(x_{d}^{*})} + \omega_{o}\left[\frac{(2-\alpha-\alpha^{*})\theta}{(1-\theta)u'(x_{d}^{*})} + 1-\alpha-\alpha^{*}\right].$$
(60)

Equation (60) states that the aggregate demand for collateral equals its aggregate supply, adjusted to reflect the balance sheet costs faced by intermediaries. Aside from (60), we can derive equilibrium conditions identical to those without an ON RRP facility, except that  $z_o = z_e = z_b$  in equilibrium. The interest rates on ON RRPs, interbank loans, and Home bonds are identical, making these assets perfect substitutes for mutual funds.

Consider an experiment where the Home central bank increases the size of the ON RRP facility  $\omega_o$  while holding constant both the size of its balance sheet  $\omega$  and the nominal interest rate on reserves (or equivalently, the price of reserves  $z_m$ ). Without a change in  $\omega$ , an increase in  $\omega_o$  implies a decrease in the total value of currency and reserves outstanding  $\bar{c} + z_m \bar{m}$ . Assuming that either  $2 - \alpha - a^*$  (the mass of mutual fund depositors in the world) or  $\theta$  (the fraction of foreign currency-denominated assets not pledgeable as collateral) is sufficiently high, this intervention effectively

<sup>&</sup>lt;sup>13</sup>From the third quarter of 2014 to the fourth quarter of 2019, the share of total reserves held by foreign banks declined from 50.2 percent to 34.2 percent, and the ratio of reserves to total assets for foreign banks fell from 56.2 percent to 26.8 percent.

increases the aggregate supply of collateral, relaxing the balance sheet constraints faced by financial intermediaries. As a result, from (42)-(45), (50), and (60), consumption quantities in all DM meetings increase.

This intervention works in the opposite way to the Home central bank's balance sheet expansion, which involves exchanging reserves for Home bonds. As noted in the previous section, a balance sheet expansion can tighten the consolidated collateral constraint in the financial system when the limited access to reserves renders them less effective as collateral compared to more widely available Home bonds. In contrast, increasing the size of the ON RRP facility involves the central bank's swaps of ON RRPs for reserves. Since mutual funds can hold ON RRPs as substitutes for interbank loans and Home bonds, the nominal interest rates on these assets rise until they align with the ON RRP rate. Therefore, this intervention effectively relaxes the consolidated collateral constraint, enabling financial intermediaries to offer welfare-enhancing deposit contracts.

#### 6.2 Case 2: Home bonds are used as medium of exchange at the margin

With the ON RRP facility, we can rewrite the consolidated collateral constraint (57) as:

$$\alpha(1-\rho)x_d \left[u'(x_d) + \frac{\delta}{1-\delta}\right] + \alpha\rho x_c \left[u'(x_c) + \frac{\delta}{1-\delta}\right]$$

$$+ \left[\alpha^*(1-\rho) + (2-\alpha-\alpha^*)(1-\pi)\right]x_d^* \left[u'(x_d^*) + \frac{\theta}{1-\theta}\right]$$

$$+ (2-\alpha-\alpha^*)\pi(1-\gamma)x_b^* \left[u'(x_b^*) + \frac{\theta}{1-\theta}\right]$$

$$+ \alpha^*\rho x_c^* u'(x_c^*) \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right]$$

$$= \omega + \omega_o \left[(2-\alpha-\alpha^*)\left\{1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right\} - 1\right] + v^* \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right]. \quad (61)$$

Other than (61), the equilibrium conditions are identical to those without an ON RRP facility, except that  $z_o = z_e$ . The ON RRP rate equals the interest rate on interbank loans in equilibrium, making ON RRPs perfect substitutes for interbank loans held by mutual funds.

Suppose the Home central bank increases the size of the ON RRP facility  $\omega_o$  while holding constant the size of its balance sheet and the nominal interest rate on reserves. Then, the aggregate supply of collateral effectively increases, assuming that either  $2 - \alpha - a^*$  or  $\theta$  is sufficiently high. As a result, from (42)-(45), (56), and (61), consumption quantities  $x_c$ ,  $x_d$ ,  $x_c^*$ ,  $x_d^*$ ,  $x_b^*$ , and  $x_{gd}$  increase.

Unlike a balance sheet expansion, an increase in the ON RRP facility alters the composition of the Home central bank's liabilities. Since the central bank only exchanges ON RRPs for reserves, the real quantity of Home bonds outstanding remains unchanged, leaving the quantity of consumption in Home bonds-involved DM meetings,  $x_b$ , unaffected, as represented by (58).

In both cases examined above, replacing reserves with ON RRPs enhances welfare for all types

of depositors—those in Home banks, Foreign banks, and mutual funds. This finding may seem counterintuitive since banks reduce their reserve holdings, which might appear to constrain their ability to engage in financial intermediation. However, the reduction of banks' reserve holdings comes with a decrease in Foreign banks' interbank borrowings and an increase in mutual funds' holdings of ON RRPs. This change relaxes the collateral constraints faced by mutual funds, lowering the liquidity premium on all collateralizable assets, including reserves. Consequently, the real return on reserves increases, allowing both Home and Foreign banks to better serve their depositors, despite holding fewer reserves.

# 7 Conclusion

Considering how monetary policy works in the United States since late 2008, it has become essential to understand the implications of foreign banks' behavior in interbank markets for U.S. monetary policy. To explore this, we have constructed a two-country model of banks and non-bank financial intermediaries that incorporates institutional differences among these financial intermediaries. The model generates active interbank markets, where foreign banks borrow from non-bank financial intermediaries, and an interest rate spread between reserves and interbank loans, consistent with data.

Given that foreign banks are active in interbank markets, an expansion of the Fed's balance sheet can have different effects depending on the stock of government bonds held by the public. Notably, the Fed's balance sheet expansion can decrease global welfare. This negative effect can occur as the Fed injects relatively poorer collateral (reserves only accessible to banks) while taking out relatively better collateral (government bonds that are widely accessible). This intervention tightens the balance sheet constraints for all financial intermediaries, leading to contractions in all financial sectors and welfare loss for both domestic and foreign consumers.

We have also examined the implications of introducing an overnight reverse repurchase agreement (ON RRP) facility on global financial markets. This facility, designed to enable non-bank financial intermediaries to earn interest on the Fed's overnight liabilities, is expected to have a more significant impact on foreign banks, given their reliance on interbank borrowing. Despite a decrease in foreign bank's arbitrage profits, increasing the size of the ON RRP facility, accompanied by a reduction in reserves, helps relax the balance sheet constraints for financial intermediaries across the global financial system, ultimately benefiting both domestic and foreign consumers.

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# A Appendix

#### A.1 Omitted Proofs

**Proof of Proposition 1:** Suppose that  $b_h \ge 0$ ,  $b_f \ge 0$ ,  $b_h^* = 0$ ,  $b_f^* \ge 0$ ,  $m_h \ge 0$ , and  $m_f \ge 0$ . Also, suppose that  $\pi \gamma x_b \le \frac{\beta b_g}{\mu}$  and  $\pi (1 - \gamma) x_b^* = \frac{\beta b_g^*}{\mu^*}$ . Then, from (15)-(33),  $z_b = z_m = z_e$ . Consequently, there are no incentives for financial intermediaries to participate in the interbank market, i.e.,  $e_h = e_f = e_g = 0$ . Using (1), (2), (5), (9), (13), and (15)-(39), we can derive an equilibrium condition as:

$$\begin{split} v + \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right] v^* \\ &\geq \alpha \rho x_c \left[u'(x_c) + \frac{\delta}{1-\delta}\right] + \alpha (1-\rho) x_d \left[u'(x_d) + \frac{\delta}{1-\delta}\right] \\ &+ \alpha^* \rho x_c^* u'(x_c^*) \left[1 + \frac{\theta}{(1-\theta)u'(x_d^*)}\right] + \alpha^* (1-\rho) x_d^* \left[u'(x_d^*) + \frac{\theta}{1-\theta}\right] \\ &+ (2-\alpha-\alpha^*) \left\{(1-\pi) x_{gd} u'(x_{gd}) + \pi \gamma x_b u'(x_b) + \pi (1-\gamma) x_b^* \left[u'(x_b^*) + \frac{\theta}{1-\theta}\right]\right\}. \end{split}$$

The above equation indicates that the aggregate demand for collateral to support consumption in DM meetings is equal to its aggregate supply in equilibrium. Suppose that financial intermediaries' balance sheet constraints do not bind in equilibrium. Then, depositors will be able to enjoy an efficient level of consumption in DM meetings with type d sellers in that  $x_d = x_d^* = x_{gd} = x^e$  where  $u'(x^e) = 1$ . This implies that from (27) and (30),  $x_b = x_b^* = x^e$ . The above constraint will be

rewritten as:

$$\begin{aligned} v + \frac{v^*}{1-\theta} \\ &\geq \alpha \rho x_c \left[ u'(x_c) + \frac{\delta}{1-\delta} \right] + \frac{\alpha^* \rho x_c^* u'(x_c^*)}{1-\theta} \\ &\quad + x^e \left[ \frac{\alpha(1-\rho)}{1-\delta} + \frac{\alpha^*(1-\rho)}{1-\theta} + \frac{(2-\alpha-\alpha^*)\left[1-\theta(1-\pi-\pi\gamma)\right]}{1-\theta} \right] \\ &\geq x^e \left[ \frac{\alpha(1-\rho)}{1-\delta} + \frac{\alpha^*(1-\rho)}{1-\theta} + \frac{(2-\alpha-\alpha^*)\left[1-\theta(1-\pi-\pi\gamma)\right]}{1-\theta} \right] \end{aligned}$$

This implies that, if v and  $v^*$  are sufficiently small so that

$$(1-\theta)v + v^* < x^e \left\{ (1-\rho) [\frac{\alpha(1-\theta)}{1-\delta} + \alpha^*] + (2-\alpha - \alpha^*) [1-\theta(1-\pi + \pi\gamma)] \right\},$$

financial intermediaries' balance sheet constraints bind,  $b_h^* = 0$ , and  $\pi (1 - \gamma) x_b^* = \frac{\beta b_g^*}{\mu^*}$ .

**Proof of Proposition 2:** First, note that from (33), mutual funds are indifferent between borrowing and lending on the interbank market. From the first-order conditions for Home and Foreign banks ((19), (20), (25), and (26)), the equilibrium price of interbank loans must satisfy the following:

$$(1-\delta)u'(x_d) + \delta \le \frac{\mu z_e}{\beta} \le u'(x_d),$$
  
$$(1-\theta)u'(x_d^*) + \theta \le \frac{\mu z_e}{\beta} \le u'(x_d^*).$$

A necessary and sufficient condition for Foreign banks to borrow on the interbank market while Home banks remain inactive is given by:

$$u'(x_d^*) < u'(x_d).$$

In equilibrium, both Home and Foreign banks hold reserves, i.e.,  $m_h > 0$  and  $m_f > 0$ . This implies that:

$$(1-\delta)u'(x_d) + \delta = (1-\theta)u'(x_d^*) + \theta.$$

Therefore, the above necessary and sufficient condition can be rewritten as:

$$\delta > \theta$$
.

**Proof of Proposition 3:** From (2), (21), (30), (37), and (38), the quantity of Foreign bonds held by Foreign banks can be expressed as:

$$b_f^* = \frac{v^* - \alpha^* \rho x_c^* u'(x_c^*) - (2 - \alpha - \alpha^*)(1 - \gamma)\pi x_b^* u'(u_b^*)}{\alpha^* z_b^*}.$$

For  $b_f^*$  to be positive, it must be the case that

$$v^* > \alpha^* \rho x_c^* u'(x_c^*) + (2 - \alpha - \alpha^*)(1 - \gamma)\pi x_b^* u'(u_b^*).$$

In this case, the prices of reserves, Foreign bonds, and interbank loans can be expressed as:

$$z_{m} = \frac{\beta[(1-\delta)u'(x_{d})+\delta]}{\mu} = \frac{\beta[(1-\theta)u'(x_{d}^{*})+\theta]}{\mu},$$

$$z_{b}^{*} = \frac{\beta u'(x_{b}^{*})}{\mu^{*}} = \frac{\beta u'(x_{d}^{*})}{\mu^{*}},$$

$$z_{e} = \begin{cases} \frac{\beta[(1-\theta)u'(x_{d}^{*})+\theta]}{\mu}, & \text{if } b_{h} \ge 0\\ \frac{\beta u'(x_{d}^{*})}{\mu}. & \text{if } b_{h} = 0 \end{cases}$$

Therefore, in equilibrium, the prices reserves, Foreign bonds, and interbank loans are positively correlated.